Physics 121 Equations

Chapter 2: Kinematic in one dimension

$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$ $v_f = v_i + a\Delta t$ $v_f^2 = v_i^2 + 2a\Delta x$	Kinematic equations for linear motion when the acceleration is constant.
$v_x = \frac{dx}{dt}$	Calculus of the velocity function.
$a_x = \frac{dv_x}{dt}$	Calculus of the acceleration function.
$x_f = x_i + \int_{t_i}^{t_f} v_x dt$	Calculus of the velocity/position functions.
$v_f = v_i + \int_{t_i}^{t_f} a_x dt$	Calculus of the velocity/acceleration function.
$a = g\sin\theta$	Acceleration of object on <u>frictionless</u> incline.
$\omega = \frac{d\theta}{dt}$	Definition of angular velocity.

Chapter 3: Vectors

$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$	Adding vectors using components.
$\lvert\vec{A}\rvert = \sqrt{A_x^2 + A_y^2}$	Magnitude of a vector.
$\theta = \tan^{-1}\frac{A_y}{A_x}$	Angle a vector makes with the x-axis.
$A_x = A\cos\theta$ $A_y = A\sin\theta$	Vector components (When θ is measured from the x-axis)

Chapter 4: Kinematic in two dimensions

$\omega = \frac{d\theta}{dt}$	Definition of angular velocity.
$\alpha = \frac{d\omega}{dt}$	Definition of angular acceleration.
$a_c = \frac{v^2}{r} = \omega^2 r$	Definition of centripetal acceleration.
$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$	Definition of period.
$\omega_f = \omega_i + \alpha\Delta t$ $\theta_f = \theta_i + \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$ $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	Kinematic equations for rotating objects when the acceleration is constant.
$s = r\theta$	Conversion from angular position to position along the arc.
$v = r\omega$	Conversion from angular velocity to tangential velocity.

$a = r\alpha$	Conversion from angular acceleration to tangential acceleration.
$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$	Relative motion conversion. (Given an object's velocity in one frame of reference, what is that object's velocity in another frame of reference)

Chapter 6: Newton's Second Law

$\vec{F}_{\text{net}} = m\vec{a}$	Newton's Second Law.
$\vec{F}_{\text{B on A}} = -\vec{F}_{\text{A on B}}$	Newton's Third Law.
$\sum F_x = 0 \quad \sum F_y = 0$	Conditions for equilibrium.
$F_G = mg$	Force of gravity
$f_{\text{s max}} = \mu_s N$	Static Friction.
$f_{\text{k}} = \mu_k N$	Kinetic Friction.
$f_{\text{R}} = \mu_R N$	Rolling Friction.
$\vec{D} = \frac{1}{2} C \rho A v^2$	Air Drag (resistance).
$v = \sqrt{\frac{2mg}{C\rho A}}$	Terminal velocity.

Chapter 8: Newton's Second Law for circular motion

$\sum F_r = ma_r = m\frac{v^2}{r} = m\omega^2 r$	Newton's Second Law in radial direction.
$\sum F_t = ma_t$	Newton's Second Law in tangential direction.
$v = \sqrt{rg}$	Speed of orbiting object

Chapter 9: Work and Energy

$W = \int_{s_i}^{s_f} F_s ds$	Work (non constant force)
$W = \vec{F} \cdot \Delta\vec{r}$	Work (constant force)
$W_{\text{net}} = \Delta K$	Work-Kinetic Energy theorem
$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}}$	Conservation of energy including work and friction
$\Delta E_{\text{th}} = f_k \Delta s$	Thermal Energy
$F_s = -\frac{dU}{ds}$	Relation between force and potential energy (conservative force)
$P = \frac{dE}{dt}$ $P = \vec{F} \cdot \vec{v}$	Power

Chapter 10: Energy

$K = \frac{1}{2}mv^2$	Kinetic Energy.
$U_g = mgy$	Gravitational Potential Energy.
$U_s = \frac{1}{2}k\Delta s^2$	Elastic Potential Energy.
$K + U_g + U_s$	Mechanical Energy.
$K_f + U_f = K_i + U_i$	Conservation of Mechanical Energy.
$F = -k\Delta s$	Hooke's Law.
$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$ $(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$	Perfectly elastic collisions

Chapter 11: Momentum

$\vec{p} = m\vec{v}$	Linear momentum.
$J = \int_{t_i}^{t_f} F(t)dt$	Impulse.
$\Delta p_x = J_x \quad \Delta p_y = J_y$	Impulse-momentum theorem.
$\vec{F} = \frac{d\vec{p}}{dt}$	Newton's Second Law.
$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2$	Conservation of momentum.

Chapter 12: Rotational Motion

$\vec{\tau} = \vec{r} \times \vec{F}$	Torque
$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots}$	Center of Mass
$x_{\text{cm}} = \frac{1}{M}\int x dm$	Center of Mass
$I = m_1 r_1^2 + m_2 r_2^2 + \ldots$	Moment of Inertia
$I = \int r^2 dm$	Moment of Inertia
$\vec{\tau} = I\alpha$	Newton's Second law for rotational situation
$\vec{\tau} = \frac{d\vec{L}}{dt}$	Newton's Second law for rotational situation
$L = \vec{r} \times \vec{p}$	Angular Momentum of collection of point masses
$L = I\omega$	Angular Momentum of solid object

$I = I_{\text{cm}} + md^2$	Parallel-axis theorem
$I = \frac{2}{5}mR^2$	Moment of Inertia for solid sphere rotated about its center
$I = \frac{2}{3}mR^2$	Moment of Inertia for a hollow sphere rotated about its center.
$I = mR^2$	Moment of Inertia for hoop or thin cylindrical shell rotated about center
$I = \frac{1}{2}mR^2$	Moment of Inertia for solid cylinder or disk rotated about its center
$I = \frac{1}{12}mL^2$	Moment of Inertia for long rod rotated about its center.
$I = \frac{1}{3}mL^2$	Moment of Inertia for long rod rotated about its end.
$K_{\text{rot}} = \frac{1}{2}I\omega^2$	Rotational kinetic energy.

Chapter 13: Gravitation

$\vec{F}_{\text{Monm}} = -\vec{F}_{\text{monM}} = \frac{GMm}{r^2}$	Force of gravity between two objects.
$v = \sqrt{\frac{GM}{r}}$	speed an orbiting object must have.
$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$	Kepler's third law.
$U = -\frac{GMm}{r}$	Gravitational potential energy.
$v = \sqrt{\frac{2GM}{r}}$	Escape speed.
$g = \frac{GM}{R^2}$	free-fall acceleration at surface of planet.

Useful Math equations

quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric functions:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Dot Product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = |A||B| \cos\alpha$ (α is the angle between the two vectors)

Cross Product: $\vec{A} \times \vec{B} = |A||B| \sin\alpha$ (α is the angle between $\vec{A}$ and $\vec{B}$. Direction given by the right-hand rule)

Constants

Mass of Earth: 5.98×10^{24} kg

Radius of Earth: 6.37×10^6 m

Free fall acceleration: $g = 9.80$ m/s^2

Gravitational constant: $G = 6.67 \times 10^{-11}$ N m^2/kg^2

Conversions

1 in = 2.54 cm

1 mile = 1.609 km
1 meter = 39.37 in
1 mph = 0.447 m/s
1 rad = $180^\circ/\pi = 57.3^\circ$
1 rev = $360^\circ = 2\pi$ rad